The most common approach to detecting and tracking a GPS signal is based on a sliding-correlator (or cross-correlator) model [1]. (The reader is reminded that in the context of this paper, we use the term sliding correlator interchangeably with cross-correlator or match filter.) In this approach, the received signal is modeled as a superposition of signals coming from several satellites, channel impairments, and receiver noise. A replica of the desired satellite signal is generated locally in the receiver. A cross-correlation of the locally generated signal with the received signal forms the cross-correlation function, which is a function of the time of arrival (TOA), \( \tau \); hence the term cross-correlator estimator. A crude measurement of \( \tau \) when multiplied by the speed of light is known as a pseudorange measurement.

A cross-correlator GPS receiver design has several advantages: (1) it is very easy to understand; (2) it is easy to implement; (3) it is of reasonable cost; and (4) it is very popular in the GPS community. Nevertheless, such a design has some disadvantages: (1) it is known to be a nonoptimal solution in the sense of maximum likelihood (it is an optimum solution only when just one GPS signal and noise are present [2, 3], but if more than one GPS signal and noise are present, which is usually the case, the cross-correlator GPS receiver is generally nonoptimal); and (2) it limits the acquisition and tracking of weak GPS signals (i.e., when multiple GPS signals are present, and for signals that are 15 dB below the strongest GPS signal or 25 dB below the receiver noise floor). In other words, a sliding-correlator GPS receiver design is a compromise between suboptimality and low cost.

Several applications (e.g., indoors, heavy foliage, obstructed line of sight) require acquisition and tracking of weak GPS signals. There are several approaches for acquiring weak GPS signals. A crude approach would be simply to increase the integration time during the acquisition process, which leads to the acquisition of weaker signals [4]. However, in the context of a sliding correlator, this approach is in general suboptimal; that is, when multiple GPS signals are present, it slows down the entire acquisition process in the sense that more time is required for the measurement to become available.
Another approach would be to apply the maximum-likelihood method to acquire weak GPS signals [1] and subject to the strongest GPS signal from the incoming (received) GPS signal and then to the second-strongest signal and so forth. This approach raises two major issues. First, it does not require any changes to the architecture of the GPS receiver; therefore, there is no advantage over the design of a GPS receiver. Second, the approach tends to increase the variance on the receiver noise by subtracting the strongest GPS signal, the second-strongest GPS signal, and so forth, thus deteriorating the signal properties, especially for acquiring weak GPS signals. The reader is reminded that the maximum-likelihood approach presented in this paper is very different from the approach discussed in [1]. The most important difference is that we formulate a maximum-likelihood function considering all the signals and noise in the environment. We neither subtract nor add the strongest GPS signal. The finer details of our approach are explained later in the paper.

Another approach is the multipath-estimating delay lock loop (MEDLL), which estimates the amplitude, delay, and phase of each multipath component using the maximum-likelihood criteria [5–7]. With this approach, however, each estimated multipath correlation function component is in turn subtracted from the measured correlation function [5]. Based on the signal model of the MEDLL receiver, we conclude that this approach is optimum if and only if one satellite signal is present; however, the MEDLL receiver is in general nonoptimum if signals from more than one satellite are present. It is not enough to say that the correlation function is optimum simply because the maximum-likelihood criterion is used. It is the maximum-likelihood function and the signal model that yield an optimum solution to the problem. In fact, based on the approach presented in this paper, the MEDLL approach can be revisited to consider the more general case. This, however, may result in a more fundamental architecture change to the MEDLL receiver.

Other approaches are the fast Fourier transform (FFT)–based acquisition techniques that use blocks of samples to establish estimates of the code and frequency in a computationally efficient manner [8, 9]. However, we are not concerned with computational efficiency. We are more concerned with an optimal solution to acquiring and tracking weak GPS signals when more than one GPS signal and noise are present.

The approach discussed in this paper is based on a maximum-likelihood estimation technique applied to the general case because this approach leads to the optimum estimation of the TOA vector, \( \tau \), and Doppler frequency vector, \( f \) [10–14]. We recently discussed a maximum-likelihood approach for acquiring weak GPS signals [15, 16]. In [15], we included only the code search, omitting the Doppler search, whereas in [16], we included both the code and Doppler searches. It is well known that the Doppler frequency for every satellite must be resolved during the acquisition process to enable resolution of the code search [15]. The literature contains several techniques for resolving the Doppler [4], such as the Tong search detector, discrete (or fast) Fourier transform (DFT) acquisition, and the Vernier approach. However, all these search techniques perform Doppler resolution for one GPS satellite at a time [4]. The technique proposed here resolves all the Doppler frequencies and the TOAs for all GPS satellites in view simultaneously.

Initially, we assume that the receiver noise is the main source of impairments. However, we are aware that there are other sources of impairment that weaken a GPS signal, such as multipath, a major source of impairment in indoor environments, and unintentional interference and jamming, which are serious threats to a GPS signal. The discussion of a maximum-likelihood GPS receiver under multipath and jamming conditions is not included in this paper because of the complexity of the problem, which will be considered in the near future.

Another aspect of a GPS receiver is the design and implementation of tracking loops. We are familiar with the design of GPS receiver tracking loops, which include the design of a delay lock loop, frequency lock loop, and phase lock loop [1, 4]. In the context of a maximum-likelihood GPS receiver, the appropriate design and implementation of tracking loops corresponding to a maximum-likelihood GPS receiver will be considered in future publications.

At this point, a maximum-likelihood GPS receiver must track at least four GPS satellites to solve for its three-dimensional (3D) position and local time. The TOA approach is the most common approach used for recursively producing a navigation solution [1, 4].

The motivation for analyzing a maximum-likelihood GPS receiver is as follows. A maximum-likelihood GPS receiver leads to (1) an overall simplified receiver architecture, as discussed later; and (2) superior performance as a result of acquiring and tracking weak GPS signals as compared with a sliding-correlator GPS receiver. Tracking performance will be considered in future publications.

For these two reasons, this paper is organized as follows. First, we present the GPS signal model in a way that considers all the signals in the environment, the channel impairments, and the receiver noise. Second, we present the theoretical performance of an acquisition process for a maximum-likelihood GPS receiver design in comparison with that of a sliding-correlator GPS receiver. Third, we present some comparative performance results for
the acquisition process of a sliding-correlator GPS receiver design and that of a maximum-likelihood GPS receiver design. In the final section, we underline the main conclusions of this research. We note that a maximum-likelihood GPS receiver can almost always acquire GPS signals that are 15–25 dB below the noise floor (or the average noise level) and therefore achieve up to an order-of-magnitude improvement relative to a sliding-correlator GPS receiver design.

SIGNAL MODEL OF A MAXIMUM-LIKELIHOOD GPS RECEIVER

Consider a maximum-likelihood GPS receiver as presented in Figure 1, which contains a GPS antenna, a single radio frequency (RF) front end and analog hardware section, a digital software and receiver hardware section for all J channels, and a display section. GPS signals coming from all visible satellites are superimposed at the entrance of a GPS receiver antenna. The RF signal is preamplified and downconverted at the intermediate frequency (IF) by means of a downconverter driven by a local clock oscillator (LO). The IF GPS waveform is amplified and sampled by means of an analog-to-digital (A/D) converter.

In this paper, we consider the noisy GPS signal model at the output of an A/D converter, which is represented by the signal and noise model. A received GPS signal at the output of the IF stage, \( x(t) \), is a combination of carrier, C/A-code, P(Y)-code, navigation data, and noise. In this paper we ignore the presence of the P(Y)-code.

Consider K GPS satellites in the sky. The GPS C/A-code waveform transmitted from a k-th GPS satellite, \( x_k(t_k) \), is given by

\[
x_k(t_k) = a_k(t_k)d_k(t_k)\text{e}^{j(\omega_0 + \theta_k)}
\]

where \( t_k \) is the time of transmission of the signal; \( a_k(t_k) \) is the spreading code at time \( t_k \), which for convenience we assume to be periodic with period \( T \); \( d_k(t_k) \) is the data bit transition for satellite \( k \), spread by the code \( a_k(t) \) at data bit index \( \left\lfloor \frac{t_k}{D} \right\rfloor \) (where \( \lfloor z \rfloor \) denotes the greatest number less than or equal to \( z \)); and \( D \) is the period when a data bit transition occurs (for the GPS case, it is equal to \( D = 20T \)). \( L_i \) is the GPS carrier frequency, \( \{L_1, L_2, \text{or} L_5\} \); \( \theta_{k0} \) is some initial phase of the signal; and the index \( k \) changes from \( \{1..K\} \). Without loss of generality, assume that \( \theta_{k0} = 0 \).

Variations of the signal amplitude are incorporated into the data bit transition \( d_k(t_k) \) for ease of notation; i.e., it is assumed that the signal amplitude is constant within one data bit transition. This is probably not a bad assumption given that the GPS signal data bit changes once every 20 ms.

We further define \( [t_k]_D \) as

\[
[t_k]_D = \left\lfloor \frac{t_k}{D} \right\rfloor
\]

Hence equation (1) can be further written as

\[
x_k(t_k) = a_k(t_k)d_k([t_k]_D)e^{j(\omega_0 + \theta_k)}
\]

To make use of all \( K \) GPS waveforms, we design a GPS receiver that must acquire, track, and demodulate at least four GPS signals coming from four visible GPS satellites to produce a navigation solution. Since the time \( t_k \) is different for every GPS satellite, we define the receiver time \( t_r \) and the relation between \( t_r \) and \( t_k \) as

\[
t_r = t_k + \zeta_k \Rightarrow t_k = t_r - \zeta_k
\]

where \( \zeta_k \) is the time of flight of the signal from the k-th GPS satellite to the receiver. Substituting equation (4) into equation (1) and considering the Doppler effect as seen by an observer, we obtain

\[
x_k(t_r - \zeta_k) = a_k(t_r - \zeta_k)d_k([t_r - \zeta_k]_D)e^{j(2\pi f_k t_r + \theta_{k0} - \psi_i)}
\]

where \( f_k \) is the Doppler frequency of the k-th GPS satellite as observed from a GPS receiver. Consider the reception of K GPS C/A-code waveforms in the presence of white Gaussian noise. Let \( x(t_r) \) denote the received GPS signal from all K GPS C/A-code waveforms in the presence of white Gaussian noise, which is given by

\[
x(t_r) = \sum_{k=1}^{K} a_k(t_r - \zeta_k)d_k([t_r - \zeta_k]_D)e^{j(2\pi f_k t_r + \theta_{k0} - \psi_i)} + \varepsilon(t_r)
\]
where \( \varepsilon(n) \) represents complex, white Gaussian noise. After the downconversion process, the received signal is given by

\[
x(t_r) = \sum_{k=1}^{K} a_k(t_r - \zeta_k)
\]

where \( \delta f_r \) and \( \delta t_r \) are the receiver’s frequency and clock errors, respectively, due to the LO.

In general, we assume that the received GPS signal is discrete-time and complex baseband. We therefore mathematically write the received signal, \( x(n) \), at discrete time index, \( n \), as

\[
x(n) = \sum_{k=1}^{K} a_k(nT_s - \zeta_k)
\]

\[
d_k(nT_s - \zeta_k)|e^{i2\pi f_k(nT_s - \zeta_k)} + \varepsilon(n)
\]

where \( T_s \) is the sample period in seconds. The length of the spreading code is written as \( L = T/T_s \) samples. We note that the parameter \( L \) is a real-valued parameter because of the presence of the Doppler effect on the code as well. Fortunately, this Doppler effect is small: at a Doppler frequency of about 1.5 kHz the code period will be 0.9999990 ms instead of 1 ms, so at a sample rate of, for example, 5 MHz, \( L \) will be 4,999.995 instead of 5,000. Over 0.1 s, this may result in loss of chip. However, if we ignore the clock errors and this effect, we have

\[
x(n) = \sum_{k=1}^{K} a_k(nT_s - \zeta_k)
\]

\[
d_k(nT_s - \zeta_k)|e^{i2\pi f_k(nT_s - \zeta_k)} + \varepsilon(n)
\]

where \( n_k \) is an integer, and \( j_k \) is an integer that takes on uniform random values \( 0,1, \ldots, 19 \), we obtain

\[
d_k(nT_s - \zeta_k) = d_k(nT_s - m_kT - \tau_k)
\]

\[
d_k(nT_s - 20n_kT - j_kT - \tau_k)
\]

\[
d_k(nT_s - j_kT - \tau_k)
\]

Also, the phase term in equation (9) can be written as

\[
2\pi f_k(nT_s - \zeta_k) = 2\pi f_k(nT_s - m_kT - \tau_k)
\]

\[
= 2\pi f_k(nT_s - m_kLT_s - \tau_k)
\]

\[
= 2\pi f_k(nT_s - \tau_k - \theta_k)
\]

where \( \theta_k \) is a random number uniformly distributed between \([0, 2\pi]\) and equal to \( \theta_k = [2\pi f_km_kT]_{2\pi} \).

Substituting the results of equations (11), (13), and (14) into equation (9) yields

\[
x(n) = \sum_{k=1}^{K} a_k(nT_s - \tau_k)
\]

\[
d_k(nT_s - j_kT - \tau_k)|e^{i2\pi f_k(nT_s - \tau_k)} + \varepsilon(n)
\]

Since the data bit \( d_k(nT_s - \zeta_k) \) will also include a complex gain associated with the channel and receiver propagation (i.e., the signal amplitude variations) and is treated as a nuisance parameter for this investigation, we have

\[
x(n) = \sum_{k=1}^{K} a_k(nT_s - \tau_k)d_k(nT_s - \zeta_k)|e^{i2\pi f_k(nT_s - \tau_k)} + \varepsilon(n)
\]

The justification of equation (16) is based on the fact that the code search is independent of the data bit transition because of the multiplicity of code periods over one data bit transition. The justification for this statement is given later in the paper.

**MAXIMUM-LIKELIHOOD ACQUISITION PROCESS**

Several techniques use the maximum-likelihood criterion for estimating code phase, carrier phase, and Doppler frequency [1, 4–7]. All of these techniques are claimed to yield an optimal solution under the assumptions of the signal model they have considered, which usually is the case of one GPS satellite signal or a GPS satellite signal and its multipath components. We know this assumption is not realistic because a GPS antenna may be able to receive up to 12 GPS signals at any time. And under this condition, all the maximum-likelihood techniques presented earlier are generally nonoptimal. In fact, it is this fundamental assumption of the signal model, together with the application of the maximum-likelihood function, that makes the technique presented in this paper more attractive and realistic and opens up the possibility of more exotic work in the next generation of GPS receivers.
The acquisition process of a maximum-likelihood GPS receiver is illustrated in Figure 2. As shown in this figure, we assume that the IF GPS signal is employed to excite a two-dimensional (2D) maximum-likelihood Doppler and code estimator. The acquisition process encompasses the maximum-likelihood detection and estimation model and the Doppler and delay offset estimation. The reader is reminded that in this paper, we use the terms Doppler estimation and Doppler search interchangeably; the same is true for delay estimation and code or code-phase search.

Next we consider the maximum-likelihood detection and estimation model, which is the first part of our novel approach, and then GPS signal acquisition, which is the second part of our approach.

Maximum-Likelihood Detection and Estimation Model

It is convenient to vectorize the notation in equation (16) by assuming that we collect M samples from each period of the spreading code. We therefore define the τk delayed and fk Doppler-shifted spreading vector by

\[ a_k(f_k, \tau_k) = \begin{bmatrix}
  a_k(f_k, n_1 T_s - \tau_k) \\
  a_k(f_k, n_2 T_s - \tau_k) \\
  \vdots \\
  a_k(f_k, n_M T_s - \tau_k)
\end{bmatrix} \tag{17} \]

where \( n_m, m = 1 \ldots M \) is a set of arbitrary M samples chosen within a single spreading-code (repetition) period. To avoid edge effects, we assume that the M samples are selected so that the \( n_m T_s - \tau_k \) values do not cross a code boundary.

Additionally, we define the noise vector, the received data vector, and the GPS satellite data bit vector as

\[ \varepsilon(q) = [\varepsilon(n_1 + qL), \varepsilon(n_2 + qL), \ldots, \varepsilon(n_M + qL)]^T \tag{18} \]

\[ x(q) = [x(n_1 + qL), x(n_2 + qL), \ldots, x(n_M + qL)]^T \tag{19} \]

\[ d(q) = [d_1(q), d_2(q), \ldots, d_K(q)]^T \tag{20} \]

Note that the index q represents the q-th occurrence of the data bit and is separated by the length of the spreading code L.

Using this notation, we write the received signal model (see equation (16)) as

\[ x(q) = \sum_{k=1}^{K} a_k(f_k, \tau_k) d_k(q) + \varepsilon(q) \tag{21} \]

Equation (21) captures the invariance of the spreading code in time T. This equation evokes the received signal commonly used in array processing, wherein \( a_k(f_k, \tau_k) \) plays the role of the “steering vector” and \((f_k, \tau_k) \) that of the “angle of arrival” [15–18]. The primary difference here, however, is that the number of data bit indices, Q, may well be smaller than the dimension of the “array,” M. Equation (21) can be further written as

\[ x(q) = A(f, \tau) d(q) + \varepsilon(q) \tag{22} \]

where the m, k-th element of the matrix \( A(f, \tau) \) is given by

\[ a_{m,k} = a_k(f_k, n_m T_s - \tau_k) \tag{23} \]

and the Doppler estimation vector \( f \) and delay estimation vector \( \tau \) are defined as

\[ f = [f_1, f_2, \ldots, f_K]^T \tag{24} \]

\[ \tau = [\tau_1, \tau_2, \ldots, \tau_K]^T \tag{25} \]

Using a matrix formulation, we can further simplify the notation by assuming that we collect processing data over Q periods (Q data bits); thus, we can write

\[ X = A(f, \tau) D + E \tag{26} \]

where we have the components of the matrices \( X, D, \) and \( E \)

\[ x_{m,q} = x(n_m + qL) \tag{27} \]

\[ d_{k,q} = d_k(q) \tag{28} \]

\[ e_{m,q} = \varepsilon(n_m + qL) \tag{29} \]

Given that the receiver's noise is complex white Gaussian, it is a simple matter to see that the maximum-likelihood objective function (or estimator), \( \rho_{ML} \) of the unknown parameter vectors \( f \) and \( \tau \) can be written as

\[ \rho_{ML} = \left\| X - A(f, \tau) D \right\|^2 = \text{tr} \left( X - A(f, \tau) D \right)^H \left( X - A(f, \tau) D \right) \]

\[ \left( X - A(f, \tau) D \right) = \text{tr} \left( G - D^H A(f, \tau)^H X \right) + D^H A(f, \tau)^H A(f, \tau) D \tag{30} \]

where we employ the Frobenius norm, which is the sum of all the square magnitudes of all the matrix elements, and where

\[ G = X^H X - X^H A(f, \tau) D \tag{31} \]

Differentiating \( \rho_{ML} \) with respect to \( D^H \) and setting the result equal to zero, we find the optimal data bit matrix to be equal to

\[ \hat{D} = [A(f, \tau)^H A(f, \tau)]^{-1} A(f, \tau)^H X \tag{32} \]
Substituting the solution of $D$ from equation (32) into equation (30), we obtain
\[
X - A(f, \tau)D = X - A(f, \tau)[A(f, \tau)^H A(f, \tau)]^{-1} A(f, \tau)^H X = (I - A(f, \tau)[A(f, \tau)^H A(f, \tau)]^{-1} A(f, \tau)^H X = P[A(f, \tau)]X
\]
where $P[A(f, \tau)]$ is called the projection operator unto the subspace formed by the columns of $A(f, \tau)$:
\[
P[A(f, \tau)] = I - A(f, \tau)[A(f, \tau)^H A(f, \tau)]^{-1} A(f, \tau)^H
\]
The projection operator matrix $P[A(f, \tau)]$ satisfies the following properties:
\[
P[A(f, \tau)] = P[A(f, \tau)]^H
\]
(Hermitian symmetry property) (35)
\[
P[A(f, \tau)]^n = P[A(f, \tau)]
\]
(integer power invariance property) (36)
for any integer $n$.

Hence, based on the Hermitian symmetry property of the operator $P[A(f, \tau)]$ given by equation (36), the maximum-likelihood estimator, $\rho_{ML}$, is written as
\[
\rho_{ML} = \text{tr}[X^H P[A(f, \tau)]^2 X] = \text{tr}[X^H P[A(f, \tau)]X]
\]
(37)
Thus far we have provided the mathematical formulation of the maximum-likelihood objective function, $\rho_{ML}$, as a function of the Doppler estimation vector, $f$, and the delay estimation vector, $\tau$. The following section provides an efficient way of computing the Doppler estimation vector, $f$, and the code-offset estimation vector, $\tau$, based on equation (37), which results in the GPS signal acquisition.

**GPS Signal Acquisition**

The structure of this algorithm is in some ways similar to that of the deterministic maximum-likelihood direction-finding algorithm proposed in [18], with the “shifted codewords” playing the role of the “steering vectors.” However, in the context of a GPS receiver, the structure of the algorithm is novel.

The maximization of equation (37) over $(f, \tau)$ can proceed via alternating directions, wherein we assume that all $(f_j, \tau_j)$, $j \neq k$ are fixed, and we then optimize over $(f_k, \tau_k)$. This process may be facilitated by the following identity for a rank one change to a projection operator
\[
P(A, b] = P(A) + P(P_1(A)b)
\]
(38)
where $P_1(A) = I - P(A) = A(A^H A)^{-1} A^H$. Let $A_k$ be the GPS satellites’ “steering” matrix $A(f, \tau)$ with the $k$-th column removed $(a_k(f_k, \tau_k))$; i.e., the relation between $A(f, \tau)$, $A_k$, and $a_k(f_k, \tau_k)$ is given by
\[
A(f, \tau) = [A_k, a_k(f_k, \tau_k)]
\]
(39)
Using this identity, we can write
\[
P(A(f, \tau)) = P(A_k) + P(P_1(A_k)a_k(f_k, \tau_k))
\]
(40)
and we can write the maximum-likelihood objective function, $\rho_{ML}$, as a function of $(f_k, \tau_k)$, while holding other $(f_j, \tau_j)$ fixed, as
\[
\rho_{ML} = \text{tr}[X^H P(A_k)X] + \text{tr}[X^H P(P_1(A_k)a_k(f_k, \tau_k)]X
\]
(41)
Only the second term in equation (41) actually depends on $(f_k, \tau_k)$; therefore, using the Hermitian symmetry property of the projection operator, we can write the second term in equation (41) as [17, 18]
\[
\rho_k(f_k, \tau_k) = \frac{a_k^H(f_k, \tau_k)P(A_k)XX^H P(A_k)a_k(f_k, \tau_k)}{a_k^H(f_k, \tau_k)P(A_k)a_k(f_k, \tau_k)}
\]
(42)
The algorithm that maximizes $\rho_k(f_k, \tau_k)$ successively over $(f_k, \tau_k)$, $k = 1..K$ repeats this process several times. Further details of this process are not disclosed because of the proprietary nature of the algorithm.

The projection operators, $P(A_1)$, can be updated by using successive applications of equation (38) and by exploiting some matrix partitioning identities. If we define the following matrix:
\[
C = (A^H A)^{-1}
\]
(43)
then we have the following results:
\[
c_{kk} = \frac{1}{a_k^H P(A_k) a_k}
\]
(44)
\[
c_k = (A_k^H A_k)^{-1} a_k - c_k \frac{a_k^H a_k}{a_k^H P(A_k) a_k}
\]
(45)
where $c_{kk}$ is the $k$-th diagonal element of $C$, and $c_k$ is the $k$-th column of $C$ with its $k$-th element removed [19]. Thus $c_k$ is a $K \times 1$ vector that does not contain the $k$-th diagonal $c_{kk}$. From this we obtain
\[
P_1(A_k) a_k = a_k - A_k c_k \frac{1}{c_{kk} A_k^H A_k}
\]
(46)
It can easily be shown that if we substitute $c_{kk}$ (given by equation (44)), and the vector $c_k$ (given by equation (45)), then equation (46) is transformed into an identity.

Equation (46) should not be used directly in equation (42) since we want to update the computation of $a_k(f_k, \tau_k)$ as we search over multiple $(f, \tau)$'s. For this reason, we consider the following way of computing $P_1(A_k)$. Define the vector
\[
v_k = P_1(A_k) a_k
\]
(47)
computed from equation (46); then from equation (40) we have the following:
\[
P(A) = P(A_k) + P(v_k)
\]
or
\[
P_1(A_k) = \frac{v_k v_k^H}{v_k^H v_k} + P_1(A)
\]
(49)
From this we conclude that
\[ P_k(A_k) = a_k(f, \tau) - AC[A^H a_k(f, \tau)] \]

\[ + \frac{v_k v_k^H a_k(f, \tau)}{v_k^H v_k} \]  

After the maximization of equation (42) is performed, \( a_k \) should be changed to \( a_k(\hat{f}_k, \hat{\tau}_k) \) for the optimal estimate. This change can be propagated back into \( C \), where it amounts to inverting a fairly sparse rank 2 change of the old \( A^H A \) matrix and can therefore be computed recursively.

To minimize the effect of Doppler frequencies over a long-duration GPS waveform, it is of interest to consider the special case wherein \( Q_{/H1005}^1 \) over a single data bit. Under this condition, \( X \) in equation (42) is an \( M_{/H1003}^1 \) vector. Initially, we assume that \( P_1(A_1) = I \), which is equivalent to the sliding-correlator (or matched filter) solution:

\[ P_1(A_k) = \frac{a_1^H(f_1, \tau_1) X X^H a_1(f_1, \tau_1)}{a_1^H(f_1, \tau_1) a_1(f_1, \tau_1)} \]  

After several iterations, we build the projection operator matrix, \( P_k(A_k) \), \( k = 1..K \), to be used in equation (42). Further details of this process are not disclosed because of the proprietary nature of the algorithm.

**SIMULATION**

**Time Delay Estimation**

By treating all the signals in the environment jointly, it is possible to greatly outperform the sliding-correlator technique, especially in situations where the satellite signals are received with widely varying powers. For a simple numerical example, consider a simulated environment wherein four satellites have been received at \(-28, -25, -20, \) and \(-10 \) dB signal-to-white noise power ratio (SNWR). The received environment is modeled at complex baseband assuming the reception of a 1 ms, 1,023 chip Gold codes from 4 GPS satellites. We also assume that a standard deviation on the GPS receiver clock error is half of the chipping period, \( T_c \). We assume further that the GPS Doppler frequency is normally distributed with zero mean and 100 Hz standard deviation.

The normalized cross-correlation coefficient for the weakest signal, at \(-28 \) dB, is shown in Figure 3 (top). The simple cross-correlator cannot detect the weakest signal at \(-28 \) dB because the strongest signal at \(-10 \) dB can jam the weakest signal; i.e., a cross-correlator receiver does not have enough dynamic range to acquire weak GPS signals. This result is in total agreement with the results reported in the literature using real data and employing either a Tong search detector or an FFT search detector for a 1 ms integration time [4]. By contrast, consider the maximum-likelihood estimator (i.e., objective function), shown in Figure 3 (bottom), can detect the weakest signal most of the time. Because the strong GPS satellite codes have been canceled, the maximum-likelihood estimator can still pick up a clear peak at the correct delay most of the time.

The normalized cross-correlation coefficient for the second-weakest signal, at \(-25 \) dB, is shown in Figure 4 (top). Because the two strongest signals at 5 and 10 dB are stronger than the second-weakest signal, it is not possible for the simple cross-correlator to detect this signal at all times. This result is in total agreement with the results reported in the literature using real data and employing either a Tong search detector or an FFT search detector for a 1 ms integration time [4].
estimator, shown in Figure 4 (bottom). Because the strong GPS satellite codes have been canceled, the maximum-likelihood estimator can still pick up a clear peak at the correct delay.

The normalized cross-correlation coefficient for the second-strongest signal, at −20 dB, is shown in Figure 5 (top). Because the strongest signal at 10 dB is stronger than the second-strongest signal, it is possible for the simple cross-correlator to detect this signal at all times. By contrast, consider the multiple-user maximum-likelihood estimator, shown in Figure 5 (bottom). For this case, the estimator can pick up a clear peak at the correct delay at all times.

The normalized cross-correlation coefficient for the strongest signal, at −10 dB, is shown in Figure 6 (top). Because this signal dominates all the other signals, it is possible for the simple cross-correlator to detect this signal at all times. Similarly, the maximum-likelihood estimator, shown in Figure 6 (bottom), can pick up a clear peak at the correct delay at all times.

**Monte Carlo Simulation**

This same receive scenario was run over 200 Monte Carlo trials with the thermal noise, delay offsets Gold sequence seeds, and Doppler frequencies randomly varied. The standard moving cross-correlator achieved a median absolute delay estimation error of {181.8, 0.063, 0.043, 0.0415} μs; it was never really able to detect the weakest signal and rarely able to detect the second-weakest signal. The median error is with respect to the truth, and therefore is attributed to the noise and the detection and estimation model. The maximum-likelihood estimator was successful in all 200 trials; however, it achieved a median absolute delay error of {86.6, 43.6, 42, 41.5} ns.

Figure 7 presents the cumulative distribution function (CDF) for estimated $\tau_1$ (i.e., the first GPS satellite) using the cross-correlator (top) and the maximum-likelihood estimator. As shown in Figure 7, if the cross-correlator is used, the probability that the error for estimating $\tau_1$ will be from −950 to 820 μs is 70 percent. On the other hand, if the maximum-likelihood estimator is used, the probability that the error for estimating $\tau_1$ will be from −88 to 0 ns is 60 percent.

Figure 8 presents the CDF for estimated $\tau_2$ (i.e., the second GPS satellite) using the cross-correlator (top) and the maximum-likelihood estimator. As shown in Figure 8, if the cross-correlator is used, the probability that the error for estimating $\tau_2$ will be from −780 to 600 μs is 20 percent. On the other hand, if the maximum-likelihood estimator is used, the probability that the error for estimating $\tau_2$ will be from −88 to 0 ns is 95 percent.
Figure 9 presents the CDF for estimated $\tau_3$ (i.e., the third GPS satellite) using the cross-correlator (top) and the maximum-likelihood estimator. As shown in Figure 9, if the cross-correlator is used, the probability that the error for estimating $\tau_3$ will be from $-88$ to $0$ ns is 98 percent. On the other hand, if the maximum-likelihood estimator is used, the probability that the error for estimating $\tau_3$ will be from $-88$ to $0$ ns is 100 percent.

Figure 10 presents the CDF for estimated $\tau_4$ (i.e., the fourth GPS satellite) using the cross-correlator (top) and the maximum-likelihood estimator. There is a 100 percent probability that for both estimators, the error for estimating $\tau_4$ will be $-87$ to $8$ ns. As indicated by the simulation results, the maximum-likelihood estimator is always superior to the standard cross-correlator. Therefore, we propose the maximum-likelihood GPS receiver as a novel and optimum approach to processing GPS signals. We remind the reader that this approach is considered optimum if all the assumptions of the signal model presented in this paper are met. If we were to expand the signal model to include multipath components, the approach presented in this paper would be nonoptimal. Even in such a case, however, our maximum-likelihood estimator could be extended to include multipath and unintentional interference and jamming.

CONCLUSIONS

We have proposed a maximum-likelihood GPS receiver for processing the received GPS signals of the L1, L2, or L5 frequencies. The maximum-likelihood GPS receiver performs a simultaneous, 2D search of both the Doppler frequencies and GPS Gold codes. The Doppler bin search size should not be more that 100 Hz. Moreover, we have identified a new approach for improving TOA estimation performance by considering this a multiuser statistical estimation problem and employing maximum-likelihood estimation techniques. A simple example has been provided, showing nearly an order-of-magnitude improvement in TOA performance. Moreover, a number of numerical procedures can be employed to reduce the computational burden of the more powerful estimation technique. It is expected that this approach can yield additional benefits in GPS performance in environments where the near–far problem limits acquisition of weak GPS signals by sliding-correlator estimation. The approach is additionally expected to yield further gains as these techniques are extended to environments containing significant multipath. Simulation
data will be validated in the near future using GPS acquisition data from the Novatel ProPack AG-G2+DB9-RT2, and the results of this work will be presented in a future publication.


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